would be the case if there were a plenum. And this is how one must reply to all similar objections, which I will skip over for the sake of brevity.

REPLY TO THE MAIN ARGUMENT

The reply to the main argument is evident.

Question 9

Is a line composed of points?

For the affirmative: A point is immediately adjacent to a point; therefore, a line is composed of points. The consequence is obvious. The antecedent is proved by taking the whole multitude of points that belong to a given line and then arguing as follows: Between the first point of this line and any other point there is either (i) some point in the middle or (ii) no point in the middle. If there is no point in the middle, then a point is immediately adjacent to a point. If there is some point in the middle, then that point is between the first point and itself, since it itself is included among all the points.

For the opposite is Aristotle in Physics 6 [1.231a24-26]. /51/

REPLY TO THE QUESTION

Here one must first give the sense of the question, which is this: Are some parts of a line indivisible? And if the question is understood in this way, I reply that no part of a line is indivisible and also that no part of any continuous thing whatsoever is indivisible.

Proof 1

This is proved as follows: If some parts of a line were indivisible, then the side of a square would be equal to the diagonal, and the diagonal would be commensurable with the side. The consequence is evident, since from any point on the diagonal one can draw a line perpendicular to the side. This is evident, since one can draw perpendicular lines from each point on the one
side to each point on the opposite side—indeed, this would be the case de acto on the posited hypothesis. And each such line is drawn through some point on the diagonal. Therefore, from each point on the diagonal there is a line that is perpendicular to the side. Therefore, if there are six points on the diagonal, there will necessarily be six points on each of the sides.

Proof 2

Again, from each point on the one side one can draw a perpendicular line to a point on the opposite side, and each of these lines will touch some point on the diagonal. But since these lines are drawn from immediately adjacent points on the one side to immediately adjacent points on the other side, the lines are uniformly distant from one another. And, as a result, two lines drawn from two immediately adjacent points on the side will touch two immediately adjacent points on the diagonal; otherwise, these lines would not be uniformly distant from one another in their entirety.

An Objection. You might object as follows: If [such] a line drawn from the side met the diagonal at right angles, then it would touch only one point on the diagonal, and another line would touch another point, and in this way the argument would establish its conclusion. But because the diagonal is longer than the side, each of the lines drawn from the one side to the other side alls obliquely on the diagonal, and it therefore touches at least two points on the diagonal. And so the argument does not establish that there are as many points on the side as there are on the diagonal. And here an example is given: If one stick is placed upon another obliquely, the first stick touches none of the second stick than it would if it were placed upon it at right angles in the manner of a cross, as is evident to the senses. This is how it is in the case under discussion.

Reply to the Objection. To the contrary, if, because the diagonal is longer than the side, a line drawn through the diagonal from the one side to the other side touches two points on the diagonal (to be sure, obliquely), then it follows that if the diagonal keeps getting longer while the length of the two sides remains always the same, then the length of the diagonal can be increased in such a way that one line drawn from the one side to the other will obliquely touch two points on the diagonal, and a second line will touch

50. The gist of this last remark seems to be that if points were distinct entities in their own right and thus (on Ockham’s reckoning) finite in number, then there would be no theoretical barrier to someone’s actually drawing all the lines in question.

51. Ockham is assuming that if lines are composed of points, then the length of a line is a function of the number of points constituting the line. Thus, lines with an equal number of points are equal in length.
three points on a longer diagonal, and a third line will touch four points on a [still longer] diagonal—and so ultimately some point on a line drawn through the diagonal will touch the diagonal over the length of a foot or a house or a mile.52 /53/

From this it follows, first, that a line drawn from the side to the diagonal will not be a straight line, since on that line the point which touches the diagonal moves higher and lower. For from the fact that (i) this point will, according to you, touch many points on the diagonal obliquely and that (ii) some of these points are higher and some lower, it follows that the point that touches them moves higher and lower.

Second, it follows that some point on the line that touches the diagonal will be in different places and positions at the same time naturally.53 For from the fact that that point touches, say, six points on the diagonal, it follows that at one and the same time it is, in its entirety, in six different positions along with those points—which is impossible.

The example concerning the stick (or any other body that touches some other body either at right angles or obliquely) is irrelevant. For when one stick falls obliquely on another stick, then many parts of the top stick touch many parts of the bottom stick that they were not previously touching. And the parts that come together and touch when the stick touches obliquely are different from the parts that come together and touch when the stick touches at right angles, as is manifestly obvious to the senses. And so it is not surprising that more parts touch when the stick touches obliquely than when it touches at right angles. In the case under discussion, however, on the line that touches the diagonal at right angles and obliquely it is always the same point [that touches]—and so the example is irrelevant.

Proof 3

Again, in support of the principal claim I argue that if some parts of a line were indivisible, then there would be equally many points in a smaller circle and a larger circle. For from each point on the larger circle a straight line can be drawn through the smaller circle to the common center of both /54/ (indeed, such a line is straight on the posited hypothesis), and these lines do not

52. That is, if we fix the length of the two sides between which the lines are being drawn while allowing the other two sides to become longer (so that the diagonal also becomes longer), then the lines in question will fall on the diagonal at increasingly oblique angles. Thus, if the original objection is correct, it follows that these lines will coincide with progressively greater numbers of points on the diagonal.

53. The term ‘naturally’ (naturaltit) here connotes that no special action on God’s part is involved.
meet except at the center. Therefore, two lines drawn from two immediately adjacent points on the larger circle will either (i) pass through two points on the smaller circle or (ii) pass through one point on the smaller circle. If the first answer is given, then I have what I set out to prove, viz., that there are as many points on the smaller circle as on the larger circle. If the second answer is given, then I argue to the contrary that the two lines in question are straight and uniformly distant from one another. Similarly, if the two lines meet at the same point on the smaller circle, then the point at which they meet on the smaller circle will exist together with two immediately adjacent points belonging to two immediately adjacent lines. And for the same reason, this point on the smaller circle can exist together with three points belonging to three lines drawn from a third, still larger, circle—and thus the point in question can exist together with a thousand points belonging to a thousand lines drawn from an immense circle which exceeds the small circle by a suitable ratio. And from this it necessarily follows that a point which in this way exists together with a hundred other points (or, if you will, the segment of the circumference corresponding to this point) will have length without a plurality of points.

An Objection. You might object as follows: The same argument holds for the case of corporeal lines that are drawn from a larger circle through a smaller circle to the center. For these lines always get closer and closer to one another. Therefore, either (i) these corporeal lines will pass through different parts on the smaller circle, in which case there will be equally many parts on both circles, or (ii) they will pass through the same part, in which case, because of the differing sizes of the circles, this part will correspond to a thousand parts and be in a thousand [different] positions. 54

Reply to the Objection. I reply that this conclusion does not follow from the diversity of those lines and circles ad infinitum. For every part on the larger circle is such that some distinct part on the smaller circle, albeit a much smaller part, corresponds to it. This is obvious to the senses if the circles are constructed. And so it does not follow that there are equally many parts of the same quantity [on the two circles].

54 In the previous case, lines drawn from immediately adjacent points on the circumference of the larger circle do not continuously come closer to one another as they approach the common center, since there is no distance less than the distance between immediately adjacent points. The present objection appeals to the fact that when we make a physical drawing to illustrate the case, the lines drawn from the circumference of the larger circle seem to get closer to one another as they approach the common center. The objector then goes on to argue that the very same problems that Ockham poses for indivisibles arise here as well. So the unstated conclusion is that on this score it makes no difference whether or not one holds that circles have indivisible parts. The same anomalies arise in either case.
If, on the other hand, a circle is composed of points, then corresponding to two immediately adjacent points on the larger circle there will be either (i) two immediately adjacent points on the smaller circle (for nothing shorter than the two points can correspond on the small circle, since both are indivisible) or (ii) the same point on the smaller circle] for both. If the first, then there will be equally many points on both circles; if the second, then the point in question will be in two different positions, as was argued above.

Further, I take that point on the smaller circle that exists together with several points of many lines. That point can be the center of another circle that can be drawn around it. The lines of this circle will not be such that they meet before they reach the center and, consequently, the point in question will not in that case exist together with several points that are distinct in position, since all the lines coming from the center can be terminated at that circle. This is proved by appeal to the immediately adjacent points that come from the center and are such that immediately adjacent points on the circle correspond to them along a straight line—and different points on the circle will correspond to these two different points. And the same will hold for all the lines that come from the center. This is evident from the fact that they are terminated at that circle; therefore, the point in question can in no way exist together with different points that are distinct in position.\textsuperscript{56}

You might object as follows: If [the point in question] is the center, then it exists together with the points of the different lines that come from it. I reply that this is true, but the point in question is always in the same position, and the points of the other lines are around the center and are terminated around that center; the center is not in the same position as they are, as it was in the other example.

\textit{Proof 4}

Furthermore, if some parts of a line were indivisible, then Zeno’s argument would hold, \textit{viz.}, that the fastest moving thing would never reach the slowest moving thing. This argument presupposes certain principles. The first is that it is always the case that a nearer part of a space is reached before a more

\textsuperscript{55} In the previous case of the corporeal circles, a proportionately shorter length on the smaller circle can correspond to a longer length on the large circle. However, on the assumption that the circles are composed of indivisible points, there is no length on the smaller circle that is shorter than two points long.

\textsuperscript{56} This argument is a \textit{reductio ad absurdum} which, beginning from the assumption that the point in question is in the same place as two points that are themselves distinct by position, yields the conclusion that the point in question is not in the same place as two points that are distinct by position.
...mote part. The second is that an indivisible is not reached in space except at an instant. From these presuppositions it follows as well that if a body were moving over such a space, then in an instant it would reach just one indivisible in the space. On this basis I argue as follows: In an instant when the slow thing acquires one indivisible in the space, what does the fastest moving thing acquire? Either (i) it acquires only one indivisible, in which case the fast thing will never reach the slow thing, or (ii) it acquires two indivisibles, and so by the second presupposition it acquires one of them before the other; hence, it does not acquire it in an instant, since in an instant there is no before.

You might object that the attributes of speed and slowness do not belong to the motion of an indivisible.

Against this: The argument is the same in all respects if it is a divisible body that is moving over a space composed of indivisibles.

You might also object as follows: This same argument of Zeno's seems to go against you when you claim that a continuum is composed of things that are forever divisible. /57/

I reply that in each instant a moving thing is in a different space; but no part of the space is primarily acquired in an instant in the sense that every part of that part is acquired in that instant. And if it were assumed [to the contrary that some part is primarily acquired], then I would say that Zeno's argument is just as necessarily conclusive against those who deny indivisibles as against those who posit them. This is evident from the fact that in the instant when the slow thing acquires that part of the space in its entirety, either (i) the fast thing acquires an equal part, in which case it will never reach the slow thing, or (ii) it acquires a larger part, in which case, because of what was presupposed above, it acquires one part of that part before acquiring another—and so it does not acquire the entire part in an instant.

And so I claim that no part of a place or other form is in its entirety acquired in an instant through motion, in such a way that each part [of that part] is acquired in the instant. Now I do grant that in each instant something is acquired—and yet a part of that acquired part was acquired first, and a part of that part was acquired first, and so on ad infinitum. And thus the slow moving thing precedes the fast thing in that space by infinitely many instants. This is manifestly obvious. Hence, it is impossible that anything should be primarily acquired in an instant through any motion; rather, each thing is primarily acquired in an interval of time, and each of its parts is acquired in an interval of time.

And, therefore, by reason of the fact that in the same interval of time the fast thing passes through more spatial parts of the same quantity than the slow thing does, one who rejects indivisibles can refute Zeno's argument.
and one who posits them cannot in any way refute the argument. Moreover, this refutation is a general one for arguments that (i) concern forms acquired through motion and (ii) are formulated in order to prove that infinitely many parts of the same quantity are acquired in a brief time. Apply the refutation and this will be evident. /58/

REPLY TO NINE ARGUMENTS FOR
THE CONTRARY POSITION

Reply to Argument 1

As regards the arguments for the contrary position, my reply to the first argument for the conclusion that there are indivisibles is that it is not impossible that there should be indivisibles, since an intelligence is indivisible, and likewise an intellective soul. But it does involve a contradiction for an indivisible to exist in a quantum, since it is impossible for either a part of a quantum or an accident of a quantum to be indivisible. Now it has just been demonstrated that no part of a quantum is indivisible. That no accident of a quantum is indivisible is proved by the arguments of certain thinkers, arguments that prove that neither the beginning nor the end of a line is an indivisible accident. And it seems to me that these arguments are conclusive. And so I claim that God cannot make an indivisible of this sort, since it involves a contradiction for such a thing to be made.

Reply to Argument 2

To the second argument one can reply in one way that if a purely spherical body and a purely flat body were produced by the divine power, then for the spherical body to touch the flat body would be impossible and would involve a contradiction. For if it did touch it, then, since it could not touch it with anything indivisible, it would have to touch it with a divisible part; and /59/ since any given part is spherical (because it is a part of a sphere), it is necessarily the case that one part of that part is higher and another part is lower. And so it is necessarily the case that there is a body between [that part and the flat body]—e.g., the air, if it touches in the air.

Alternatively (and perhaps this is better), one can reply that the spherical body touches the flat body with some divisible part of itself. And when it

57. These arguments are found in Chatton, Reportatio 2, dist. 2, ques. 3, arts. 1 and 4, though he describes the first six as 'common arguments'.

58. In Summa Logicae 1, chap. 4 (OP 1:132–34). Ockham himself argues that a point is not an indivisible accident that has a quantified substance as its primary subject.
is claimed that the part in question is not spherical, I reply by denying the consequence. For this does not follow unless the given part in its entirety touches the flat body primarily in the sense that each part of that part touches the flat body—since in that case the argument would necessarily establish that the [first] body is not purely spherical. I, however, maintain that the spherical body does not touch the flat body primarily with a part that is such that each of its part touches the flat body. Therefore, it does not touch it primarily with some part that is prior to all the other touching parts. Rather, any given touching part is still such that a half of it does not touch immediately, and a half of that half does not touch immediately, and so on ad infinitum.

Reply to Argument 3

To the next argument I reply that the difference between that which is continuous and that which is contiguous is that the parts of that which is continuous constitute a single thing, whereas the parts of that which is contiguous do not.

Reply to Argument 4

In reply to the next argument I grant that any continuum has as many parts of the same ratio as the heavens have, but it does not have as many parts of the same quantity.\textsuperscript{59}

You might object as follows: Take some part of the heavens that has the same quantity as a grain of the millet plant. Then that part of the heavens has as many parts of the same quantity and as many parts of the same ratio as the grain. But the whole of the heavens \textsuperscript{60} has more parts of the same ratio than does that tiny part of it; otherwise, the whole would not be greater than its part.

I reply that the proposition 'The whole has more parts than a part does' can be understood in two ways. In one way, it means that the part has some set number of parts and that the parts of the whole exceed this number by some set amount. And this reading is false, since it implies that the part has a set number of parts—which is false.

In the second way it means that (i) a part of the heavens is not such that it has a set number of parts and no more, since it has infinitely many, and (ii) the whole of the heavens has that many parts and still other parts. And

\textsuperscript{59} What Ockham means is that any continuum, even a very short line, has infinitely many parts ordered in such a way that each is half as long as its predecessor. However, it does not follow that the line is as long as the heavens, since for any given length \( l \) that is shorter than or equal to the line, it is not the case that the line has as many parts of length \( L \) as the heavens do.
in this sense I grant that the whole of the heavens has more parts than a
part does.

Reply to Argument 5
To the next argument I reply that if the world had existed from eternity and
if God had perpetually made one division of a continuum every hour, then
the division would not yet be complete.
And when it is claimed that there would not have been more divisions
than there were instants of past time, I reply as I replied above concerning
the plurality [of parts in the heavens].

Reply to Argument 6
To the next argument I reply that we measure things, as much as we can
in our present state, by means of a divisible minimum, which is sometimes
a minimum by nature and sometimes a minimum by convention, as is
evident in the case of the ell. And this is what the Philosopher says in Metaphysics 4.

Reply to Argument 7
As for the next argument, the one about the Blessed Virgin, my reply is that
she could not have been subject to original sin for just an instant. This will
become evident later.

Reply to Argument 8
To the next argument I reply that God cannot make two angels who exist only
for an instant; rather, it is necessary that a temporal interval should elapse.

Reply to Argument 9
To the next argument I reply that the visions of whiteness succeed one
another continuously, just as the form increases continuously. And so I claim
that just as it is not necessary to posit such indivisibles because of a motion of
augmentation, so neither is it necessary to posit them because of the visions.

REPLY TO THE MAIN ARGUMENT
As for the main argument, it should be resolved by appeal to [a fallacy] of a
figure of speech. For the proposition 'Between the first point and any other
point there is a point in the middle' should be granted, but it should be
denied that there is some point that is in the middle between the first point
and any other point.
in this sense I grant that the whole of the heavens has more parts than a part does.

*Reply to Argument 5*

To the next argument I reply that if the world had existed from eternity and if God had perpetually made one division of a continuum every hour, then the division would not yet be complete.

And when it is claimed that there would not have been more divisions than there were instants of past time, I reply as I replied above concerning the plurality [of parts in the heavens].

*Reply to Argument 6*

To the next argument I reply that we measure things, as much as we can in our present state, by means of a divisible minimum, which is sometimes a minimum by nature and sometimes a minimum by convention, as is evident in the case of the ell. And this is what the Philosopher says in *Metaphysics* 4.

*Reply to Argument 7*

As for the next argument, the one about the Blessed Virgin, my reply is that she could not have been subject to original sin for just an instant. This will become evident later.

*Reply to Argument 8*

To the next argument I reply that God cannot make two angels who exist only for an instant; rather, it is necessary that a temporal interval should elapse.

*Reply to Argument 9*

To the next argument I reply that the visions of whiteness succeed one another continuously, just as the form increases continuously. And so I claim that just as it is not necessary to posit such indivisibles because of a motion of augmentation, so neither is it necessary to posit them because of the visions.

**REPLY TO THE MAIN ARGUMENT**

As for the main argument, it should be resolved by appeal to [a fallacy] of a figure of speech. For the proposition 'between the first point and any other point there is a point in the middle' should be granted, but it should be denied that there is some point that is in the middle between the first point and any other point.