a certain class of investigation. If a book in the form of a
cylindrical roll is cut by a plane and then unrolled, why is it
that the cut edge appears as a straight line if the section
is parallel to the base (i.e. is a right section), but as a crooked
line if the section is obliquely inclined (to the axis). The
Problems are not by Aristotle; but, whether this one goes
back to Aristotle or not, it is unlikely that he would think of
investigating the form of the curve mathematically.

c. The continuous and the infinite.

Much light was thrown by Aristotle on certain general
conceptions entering into mathematics such as the 'continuous'
and the 'infinite'. The continuous, he held, could not be
made up of indivisible parts; the continuous is that in which
the boundary or limit between two consecutive parts, where
they touch, is one and the same, and which, as the name
itself implies, is kept together, which is not possible if the
extremities are two and not one. The 'infinite' or 'un-
limited' only exists potentially, not in actuality. The infinite
is so in virtue of its endlessly changing into something else,
like day or the Olympic games, and is manifested in different
forms, e.g. in time, in Man, and in the division of magnitudes.
For, in general, the infinite consists in something new-being
continually taken, that something being itself always finite
but always different. There is this distinction between the
forms above mentioned that, whereas in the case of magnitudes
what is once taken remains, in the case of time and Man it
passes or is destroyed, but the succession is unbroken.
The case of addition is in a sense the same as that of division;
in the finite magnitude the former takes place in the converse
way to the latter; for, as we see the finite magnitude divided
ad infinitum, so we shall find that addition gives a sum
tending to a definite limit. Thus, in the case of a finite
magnitude, you may take a definite fraction of it and add to
it continually in the same ratio; if now the successive added
terms do not include one and the same magnitude, whatever
it is [i.e. if the successive terms diminish in geometrical
progression], you will not come to the end of the finite
magnitude, but, if the ratio is increased so that each term
does include one and the same magnitude, whatever it is, you
will come to the end of the finite magnitude, for every finite
magnitude is exhausted by continually taking from it any
definite fraction whatever. In no other sense does the infinite
exist but only in the sense just mentioned, that is, potentially
and by way of diminution. And in this sense you may have
potentially infinite addition, the process being, as we say, in
a manner the same as with division ad infinitum; for in the
case of addition you will always be able to find something
outside the total for the time being, but the total will never
exceed every definite (or assigned) magnitude in the way that,
in the direction of division, the result will pass every definite
magnitude, that is, by becoming smaller than it. The infinite
therefore cannot exist, even potentially, in the sense of exceed-
ing every finite magnitude as the result of successive addition.
It follows that the correct view of the infinite is the opposite
of that commonly held; it is not that which has nothing
outside it, but that which always has something outside it. Aristotle is aware that it is essentially of physical magnitudes
that he is speaking; it is, he says, perhaps a more general
inquiry that would be necessary to determine whether the
infinite is possible in mathematics and in the domain of
thought and of things which have no magnitude.

'But', he says, 'my argument does not anyhow rob
mathematicians of their study, although it denies the existence
of the infinite in the sense of actual existence as something
increased to such an extent that it cannot be gone through
(ad infinitum); for, as it is, they do not even need the infinite
or use it, but only require that the finite (straight line) shall
be as long as they please. . . . Hence it will make no difference
to them for the purpose of demonstration.'

The above disquisition about the infinite should, I think,
be interesting to mathematicians for the distinct expression
of Aristotle's view that the existence of an infinite series the
terms of which are magnitudes is impossible unless it is
convergent and (with reference to Riemann's developments)
that it does not matter to geometry if the straight line is not
infinite in length provided that it is as long as we please.

1 Probl. xvi. 6. 914 a 25. 2 Phys. v. 3. 227 a 11; vii. 1. 231 a 24.
Aristotle’s denial of even the potential existence of a sum of magnitudes which shall exceed every definite magnitude was, as he himself implies, inconsistent with the lemma or assumption used by Eudoxus in his method of exhaustion. We can, therefore, well understand why, a century later, Archimedes felt it necessary to justify his own use of the lemma:

‘the earlier geometers too have used this lemma: for it is by its help that they have proved that circles have to one another the duplicate ratio of their diameters, that spheres have to one another the triplicate ratio of their diameters, and so on. And, in the result, each of the said theorems has been accepted no less than those proved without the aid of this lemma.’

(4) *Mechanics.*

An account of the mathematics in Aristotle would be incomplete without a reference to his ideas in mechanics, where he laid down principles which, even though partly erroneous, held their ground till the time of Benedetti (1530-90) and Galilei (1564-1642). The *Mechanica* included in the Aristotelian writings is not indeed Aristotle's own work, but it is very close in date, as we may conclude from its terminology; this shows more general agreement with the terminology of Euclid than is found in Aristotle’s own writings, but certain divergences from Euclid’s terms are common to the latter and to the *Mechanica*; the conclusion from which is that the *Mechanica* was written before Euclid had made the terminology of mathematics more uniform and convenient, or, in the alternative, that it was composed after Euclid’s time by persons who, though they had partly assimilated Euclid’s terminology, were close enough to Aristotle’s date to be still influenced by his usage. But the Aristotelian origin of many of the ideas in the *Mechanica* is proved by their occurrence in Aristotle’s genuine writings. Take, for example, the principle of the lever. In the *Mechanica* we are told that,

\[ A \text{ will move } \frac{1}{2} B \text{ over the distance } 2 C \text{ in the time } D, \]

\[ A \text{ will move } \frac{1}{3} B \text{ over the distance } 3 C \text{ in the time } D; \]

namely that the line which is farther from the centre describes the greater circle, so that, if the power applied is the same, that which moves (the system) will change its position the more, the farther it is away from the fulcrum.’

The idea then is that the greater power exerted by the weight at the greater distance corresponds to its greater velocity. Compare with this the passage in the *De caelo* where Aristotle is speaking of the speeds of the circles of the stars:

\[ \text{it is not at all strange, nay it is inevitable, that the speeds of circles should be in the proportion of their sizes.} \]

Since in two concentric circles the segment (sector) of the outer cut off between two radii common to both circles is greater than that cut off on the inner, it is reasonable that the greater circle should be carried round in the same time.’

Compare again the passage of the *Mechanica*:

\[ \text{what happens with the balance is reduced to (the case of the) circle the case of the lever to that of the balance, and practically everything concerning mechanical movements to the case of the lever. Further it is the fact that, given a radius of a circle, no two points of it move at the same speed (as the radius itself revolves), but the point more distant from the centre always moves more quickly, and this is the reason of many remarkable facts about the movements of circles which will appear in the sequel.} \]

The axiom which is regarded as containing the germ of the principle of virtual velocities is enunciated, in slightly different forms, in the *De caelo* and the *Physics*:

\[ \text{A smaller and lighter weight will be given more movement if the force acting on it is the same} \]

... The speed of the lesser body will be to that of the greater as the greater body is to the lesser.’

\[ \text{If } A \text{ be the moving, } B \text{ the thing moved, } C \text{ the length through which it is moved, } D \text{ the time taken, then} \]

\[ \text{A will move } \frac{1}{2} B \text{ over the distance } 2 C \text{ in the time } D, \]

\[ \text{A will move } \frac{1}{3} B \text{ over the distance } 3 C \text{ in the time } D; \]

thus proportion is maintained.’

1 *Mechanica*, 3. 850 b 1. 2 *De caelo*, ii. 8. 239 b 15. 3 *Mechanica*, 848 a 11. 4 *De caelo*, iii. 2. 301 b 4, 11. 5 *De caelo*, iii. 2. 301 b 4, 11. 6 Phys. vii. 5. 249 b 30-250 a 4.